

# Hilbert $H^*$ -modules

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The collection of sources below is far from being complete, but nevertheless, instructive as guide into the research field.

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